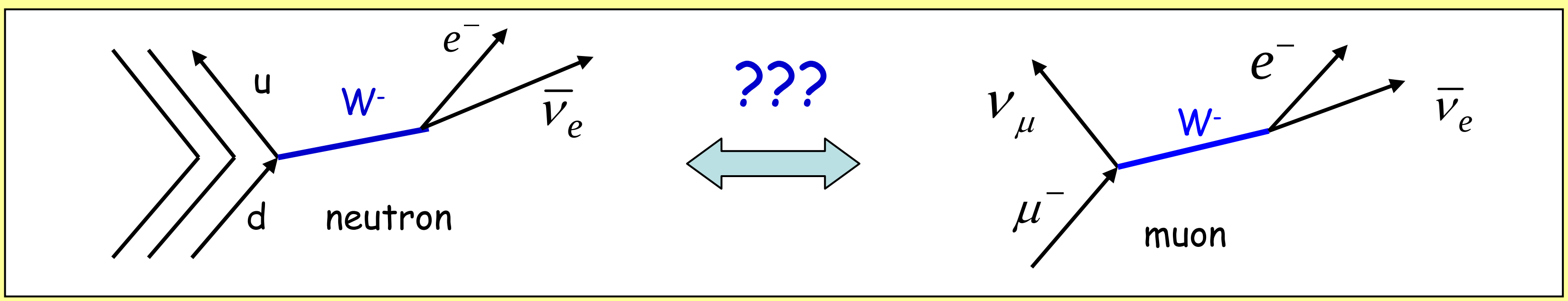
We have, so far two coupling constants for (nuclear) beta decay: G_V (S=0 decays) and G_A (S=1). These set the overall scale of the interaction, with G_V determined from the transition rates for "superallowed" 0+ → 0+ nuclear decays, and G_A from Gamow-Teller decays (0+ → 1+ and vice versa).

Other related processes:

1. **muon decay:** $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \quad or \quad \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$

- a "purely leptonic" weak decay -- no quarks before or after!
- no change of electric charge; must be "mediated" by the neutral Z^0 boson
- no "Fermi function" needed, since no Coulomb effects in the final state.
- analogous to neutron decay, so we can try the same formalism, assuming weak interactions for quarks and leptons are the same



Muon lifetime implications:

$$\mu^{\pm} \rightarrow e^{\pm} + \nu_e / \overline{\nu}_e \; + \overline{\nu}_\mu / \nu_\mu$$

measured lifetime: $\tau = 2.19703 \pm 0.00004\ \mu s$

theoretical prediction:

$$\tau = \frac{192\ \pi^3\ \hbar^7}{G^2\ m_\mu^5\ c^4}$$

(our prediction, integrated over phase space for the two neutrino types!)

Muon decay gives a weak coupling constant G that is about 2.5% larger than in nuclear beta decays....

or alternatively, the coupling constant for the $d \rightarrow u$ quark weak transition is about 2.5% smaller than that for the $\mu \rightarrow e$ lepton weak transition.

2. Pion decay: $\pi^- \rightarrow \pi^0 + e^- + \overline{\nu}_e$

Another d → u quark transition; rate is consistent with the same coupling constants as nuclear beta decay

3. K meson decay $K^- \rightarrow \pi^0 + e^- + \overline{\nu}_e \qquad (\overline{u}s \rightarrow \overline{u}u + e^- + \overline{\nu}_e)$

This is an s → u quark transition; rate is much smaller than the equivalent d → u rate; coupling constants are reduced to about 20% of nuclear beta decay values ....

Weak interactions of quarks:

- There are hundreds of examples of weak decays in nuclear and particle physics.
- Purely leptonic rates are all consistent with a single weak coupling constant G
- Hadronic rates, involving quark transitions, occur at a comparable scale but with consistent differences that depend on the type of quarks involved.
- A simple pattern emerges if we assume that the quarks that participate in weak interactions are linear combinations of the strong interaction eigenstates, represented by a unitary matrix called the CKM (Cabbibo-Kobayashi-Maskawa) matrix:

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \times \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

weak eigenstates

Unitary matrix, like a rotation matrix – preserves “length”

strong eigenstates

$$d' = V_{ud}\, d + V_{us}\, s + V_{ub}\, b, \quad etc...$$

How does this work?

- Instead of a d → u transition in neutron beta decay, only the contribution from the weak eigenstate d′ plays a role, and the weak coupling constant is effectively reduced by a factor $V_{ud} = 0.974$.
- Similarly, instead of an s → u transition in kaon decay, we have an s′ → u transition, effectively reducing the weak coupling constant by a factor $V_{us} = 0.220$.
- Studies of a large number of particle decays and beta transitions have effectively "mapped out" the CKM matrix as follows: (Particle Data Group , 2004)

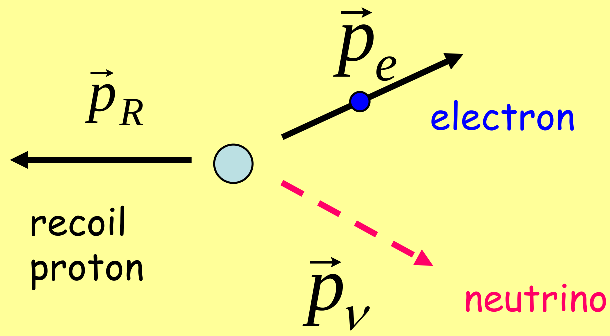
$$\left| \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \right| = \begin{bmatrix} 0.974 & 0.220 & 0.004 \\ 0.224 & 0.996 & 0.041 \\ 0.009 & 0.041 & 0.999 \end{bmatrix}$$

0.9739 to 0.9751	0.221 to 0.227	0.0029 to 0.0045
0.221 to 0.227	0.9730 to 0.9744	0.039 to 0.044
0.0048 to 0.014	0.037 to 0.043	0.9990 to 0.9992

2σ limits

- Diagonal terms dominate the CKM matrix
- All "large" terms are real; small imaginary component in lower right 2x2 submatrix allows for time reversal, or alternatively "CP violation" -- a hot research topic!

Finally, what can we learn from neutron decay?

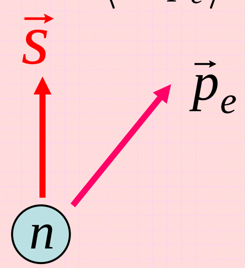


measuring spin-momentum correlations for the decay of polarized neutrons yields additional information (neutron spin: $\vec{s}$)
→ correlation coefficients: a , A, B:

$$A = -2\frac{-G_A G_V + G_A^2}{G_V^2 + 3G_A^2}, \qquad \tau = \frac{\text{constant}}{G_V^2 + 3G_A^2}$$

$$\lambda_{if} \propto p_e E_e \left(Q - E_e\right)^2 \left[1 + a\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \left\langle \vec{s} \right\rangle \cdot \left(A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu} \right)\right] dE_e d\Omega_e d\Omega_\nu$$

β asymmetry A, the correlation $\left\langle \vec{s} \bullet \vec{p}_e \right\rangle$



$$\vec{n} \rightarrow p + e^- + \overline{\nu}_e$$

"little a" and "B" are hard to measure because one cannot determine the neutrino momentum directly.

The best additional measurement is the "big A" coefficient, which gives an independent constraint from the neutron lifetime, but one has to control and measure the neutron spin direction and measure the electron momentum / energy very precisely...

new experiment with ultra cold neutrons:

http://www.krl.caltech.edu/ucn/

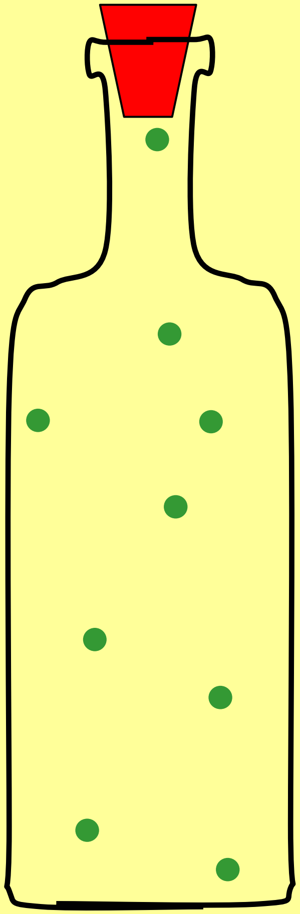
What are Ultra Cold Neutrons (UCN) ?

- UCN are neutrons that are moving so slowly that they are totally reflected from a variety of materials.

- They can be confined in material bottles for long periods of time.

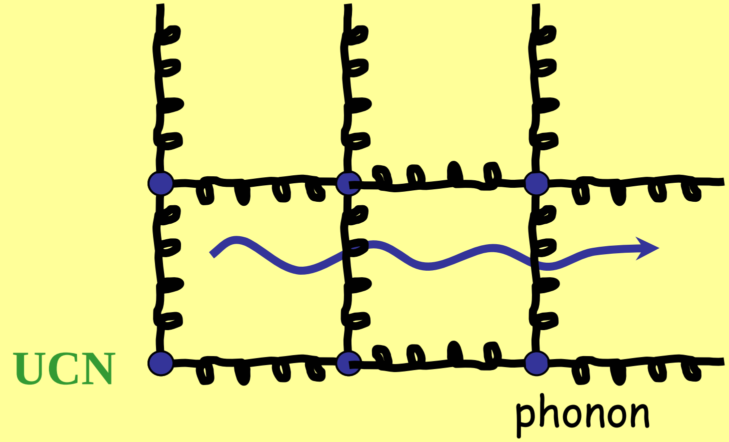
- Typical parameters:
  - velocity < 8 m/s
  - temperature < 4 mK
  - kinetic energy < 300 neV

- Interactions:
  - gravity: $V=mgh$
  - weak interaction (allows UCN to decay)
  - magnetic fields: $V=-\mu\bullet B$
    (100 % polarization by passing through a magnet !)
  - strong interaction

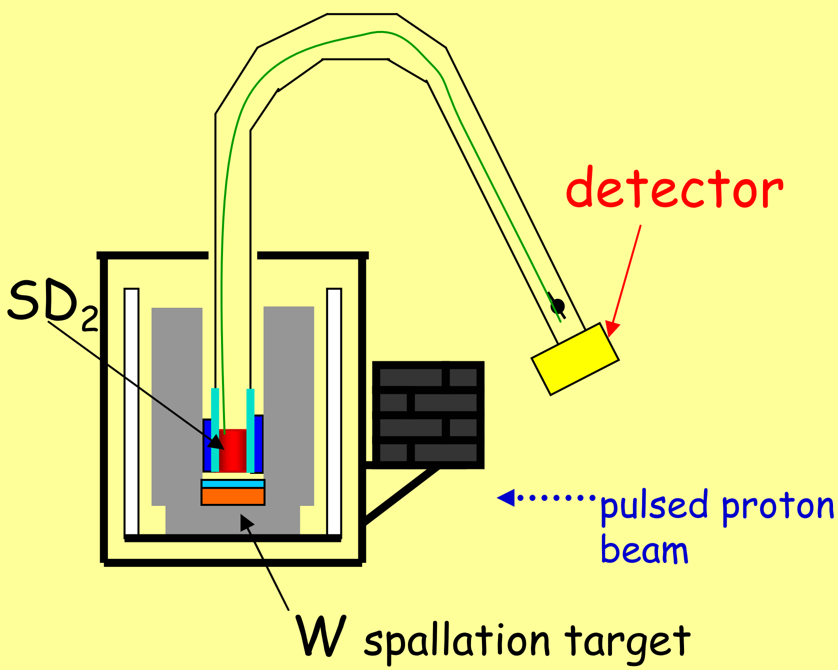


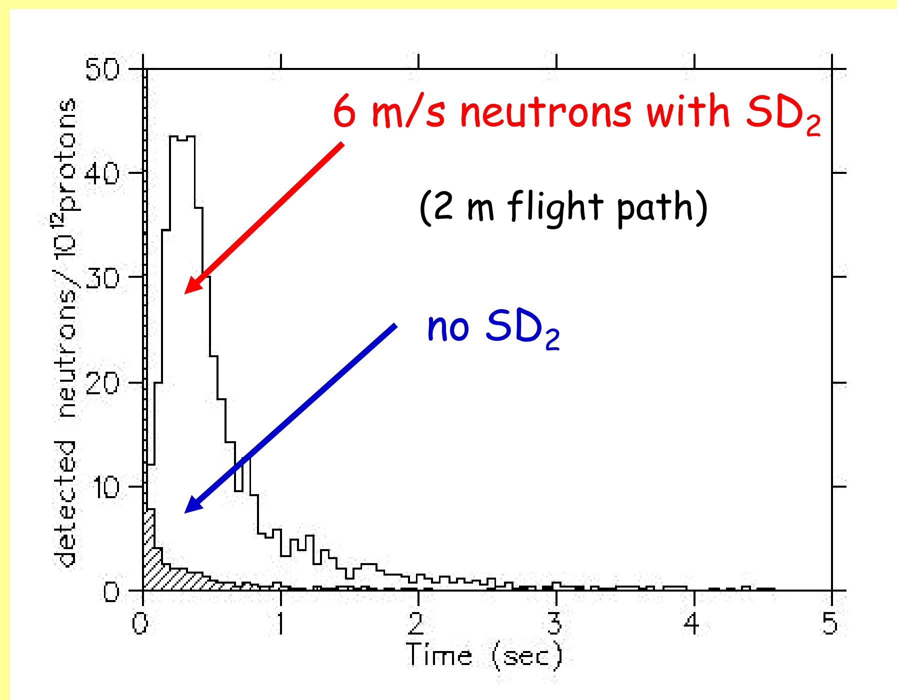
slides courtesy Prof. J. Martin, U. Wpg.

Cold n



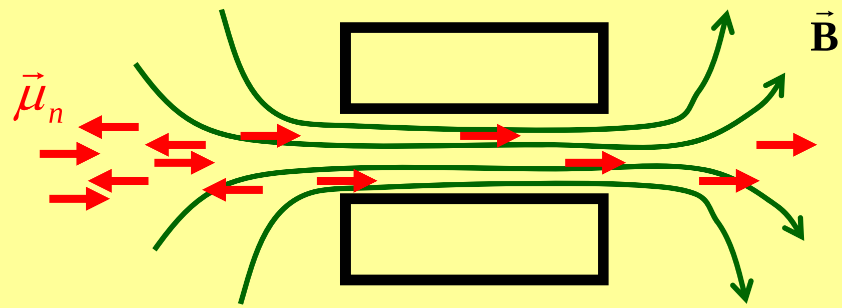
First UCN generation at Los Alamos:





UCNA Advantages: Polarization and Background

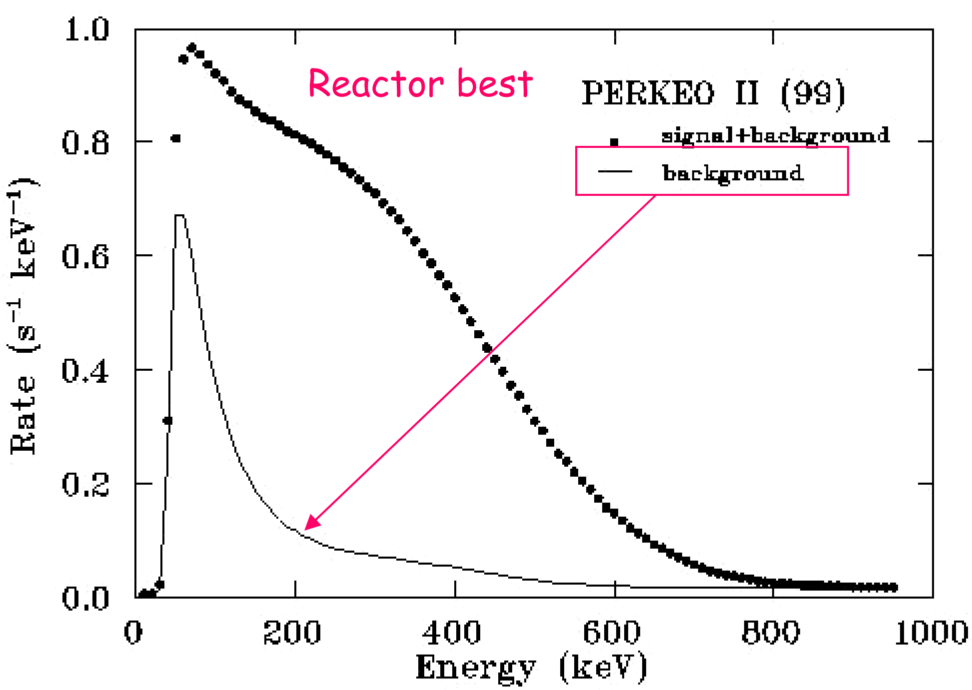
Ultra cold neutrons with the wrong spin direction can't make it through a large magnetic field!

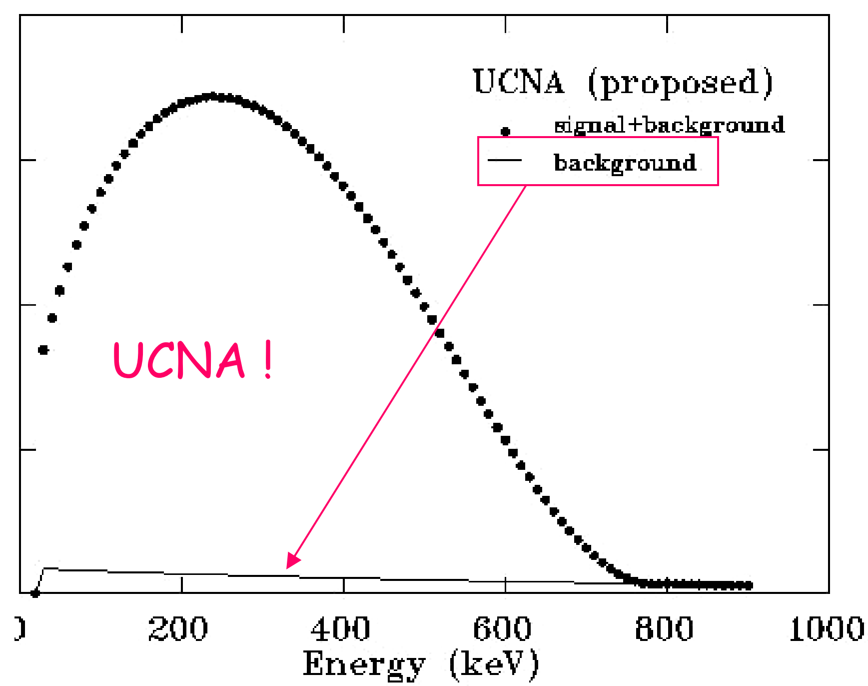


B > 6 T
→ P = 100%

limitation – magnetic scattering from walls, etc.

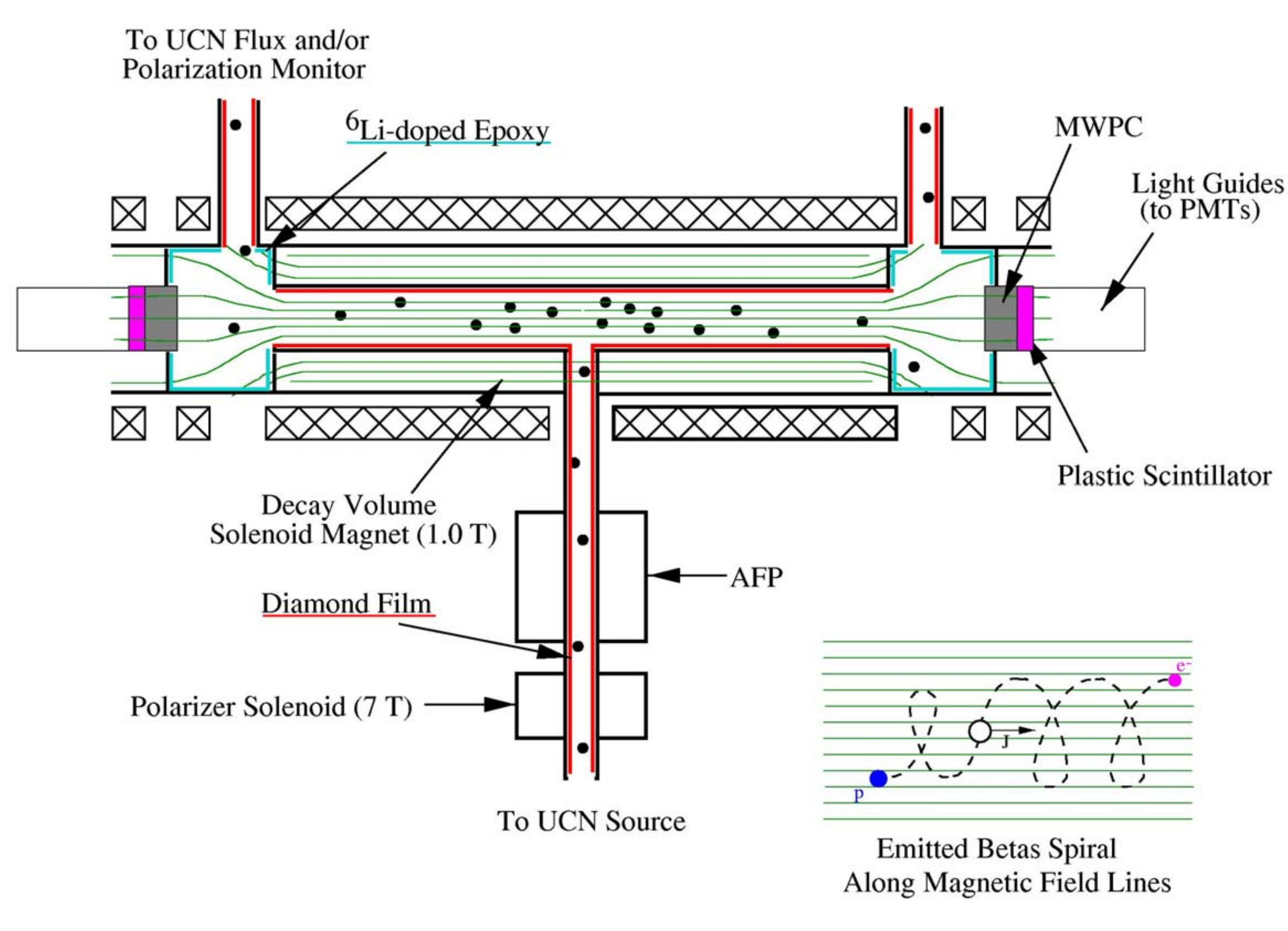
Background reduction via pulsed source:





UCNA Experiment Design

recall: correlation $< \vec{s} \bullet \vec{p}_e > / E_e$



Issues: electron backscattering from detector surface (similar issue in lifetime expt.)
neutron depolarization by scattering from the walls (<~ 0.1 %)

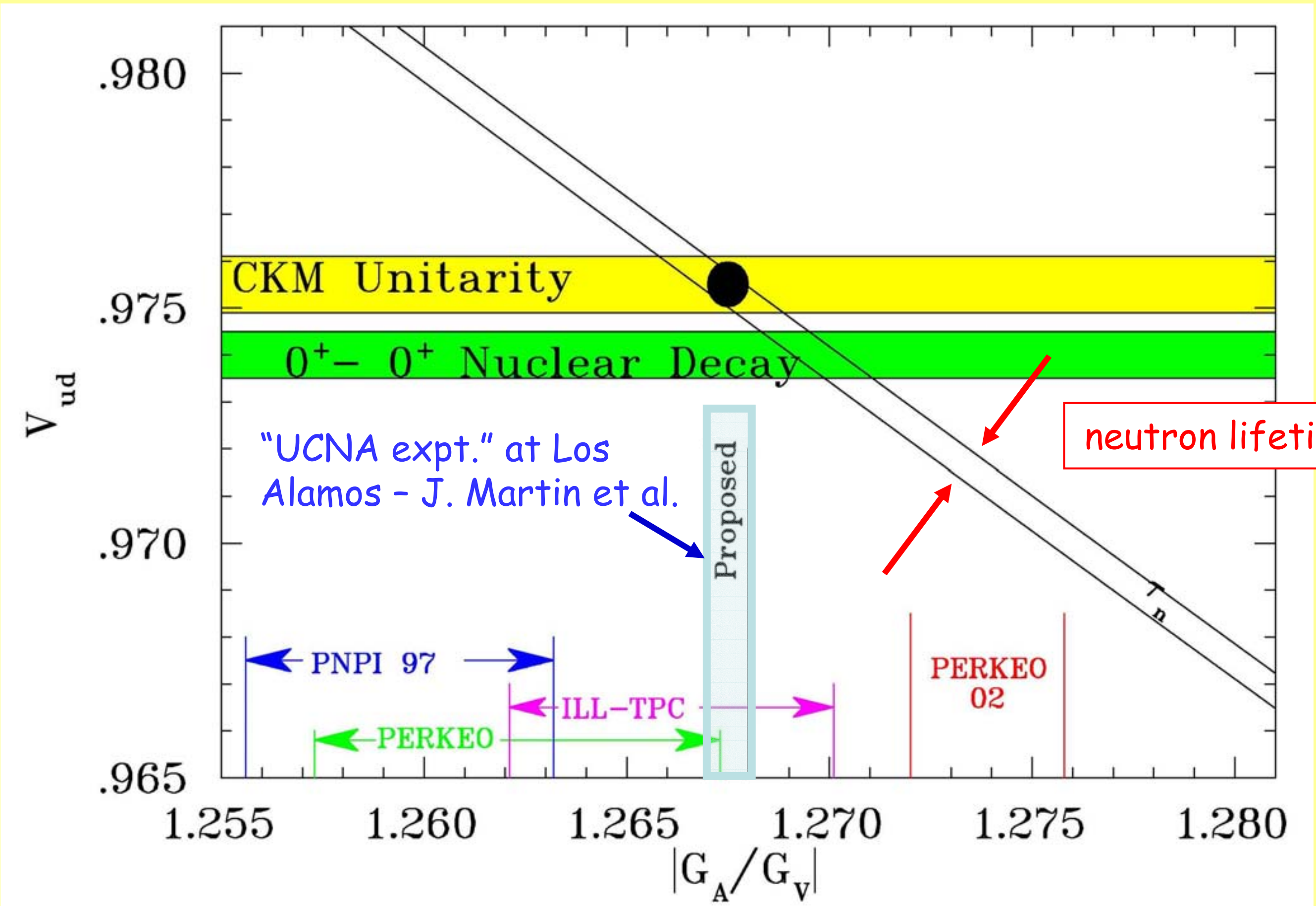
CKM Unitarity test: are there more than three generations of quarks?

$$V_{ij}^{-1} = V_{ij}^{*} \rightarrow V_{ud}^{2} + V_{us}^{2} + V_{ub}^{2} = 1 \ \ ?$$

(the best-tested row of V_{ij})

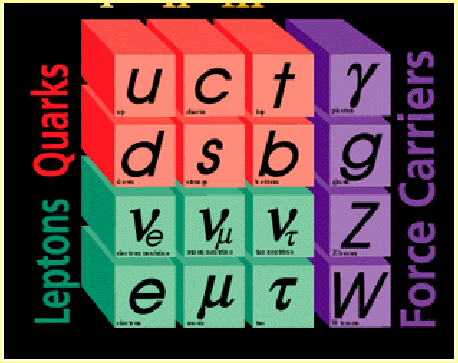
world data, 2004: $\boxed{V_{ud}^{2} + V_{us}^{2} + V_{ub}^{2} = 0.9967 \pm 0.0014}$

2σ discrepancy: contentious issue...

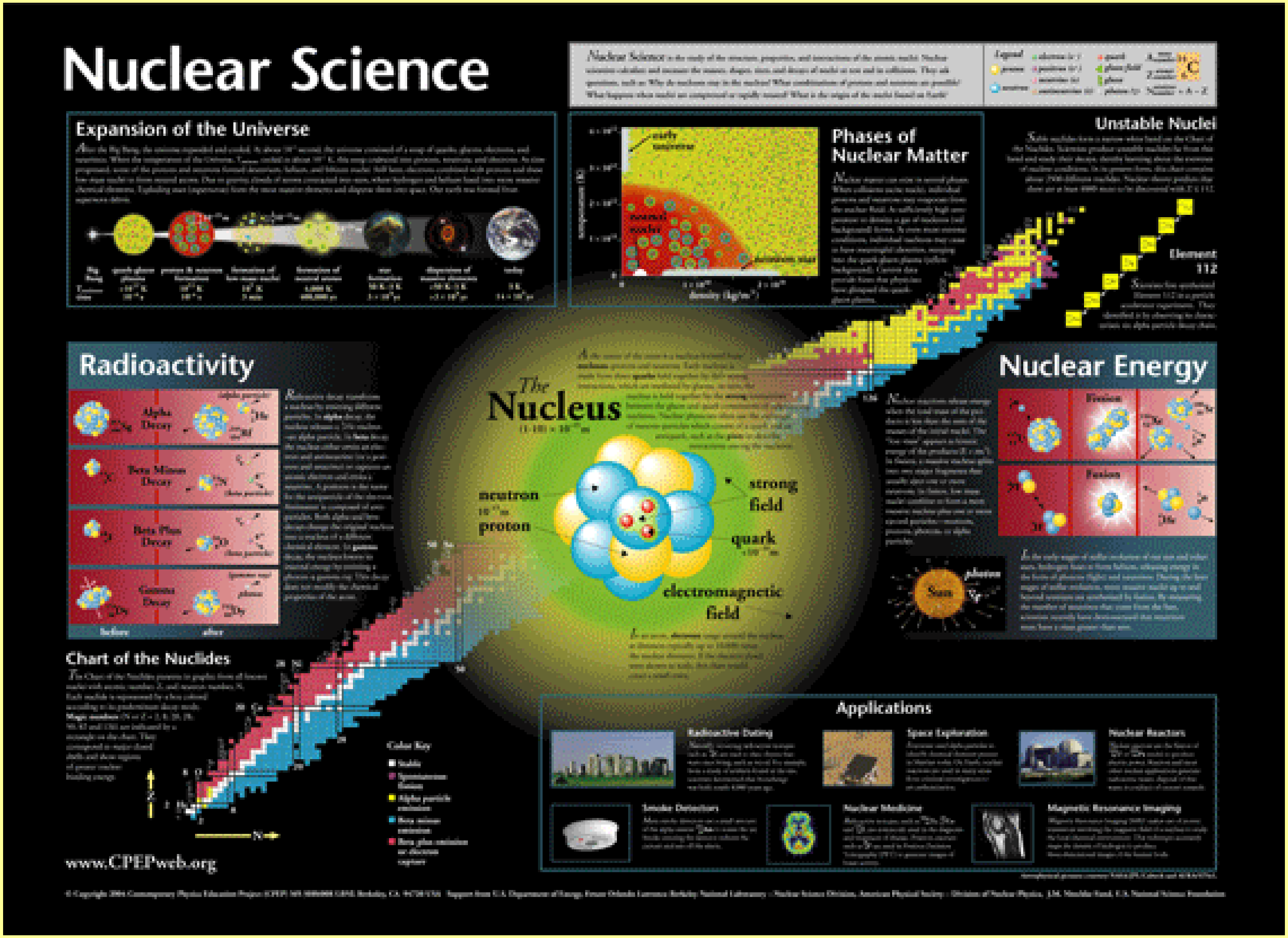


horizontal scale – measurements of "A" in neutron decay

an additional generation of quarks is not entirely ruled out by the CKM unitarity test at the present time!



Change of gears – finite nuclei:



Basic properties: $d \equiv {}^{2}_{1}\mathrm{H}$

mass: mc^2 = 1876.124 MeV

binding energy: $B \equiv \sum_i m_i - M = m_p + m_n - m_d = 2.2245731 \;\text{MeV}$

(measured via γ-ray energy in n + p $\rightarrow$ d + γ)

RMS radius: 1.963 ± 0.004 fm (from electron scattering)

quantum numbers: $J^\pi, I = 1^+,\; 0$ (lectures 13, 14)

magnetic moment: μ = + 0.8573 μ_N

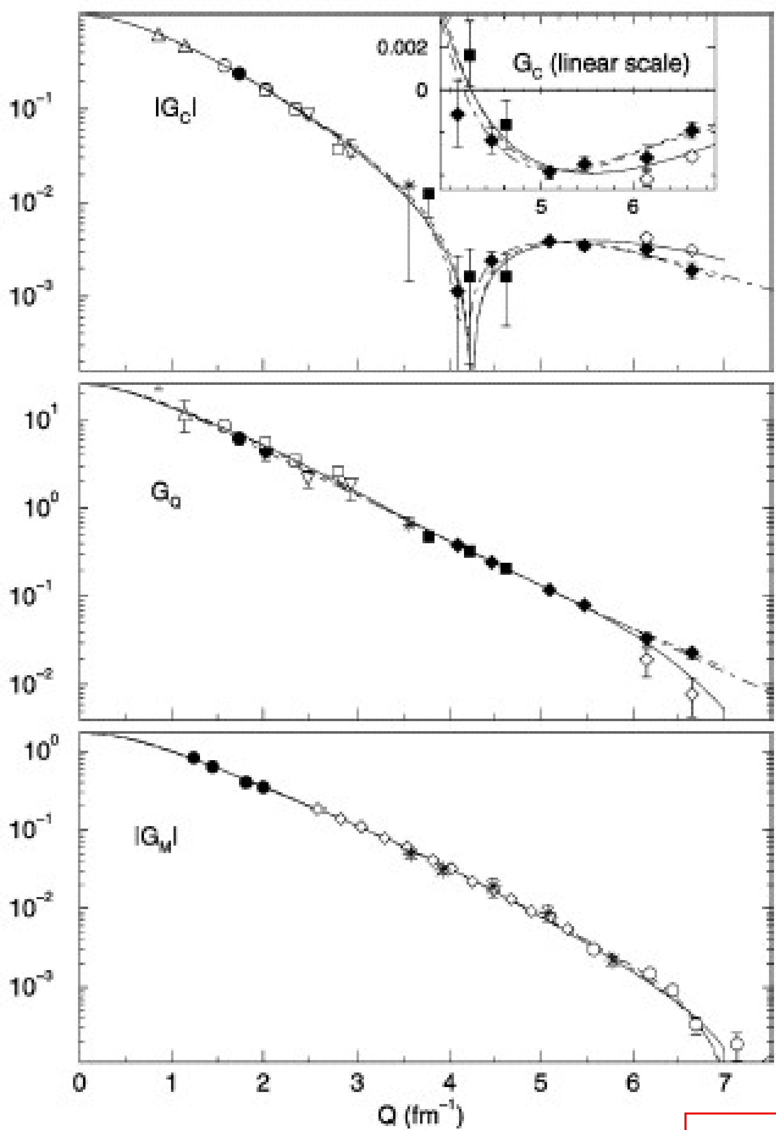
electric quadrupole moment: Q = + 0.002859 ± 0.00030 bn

($\rightarrow$ the deuteron is not spherical! ....)

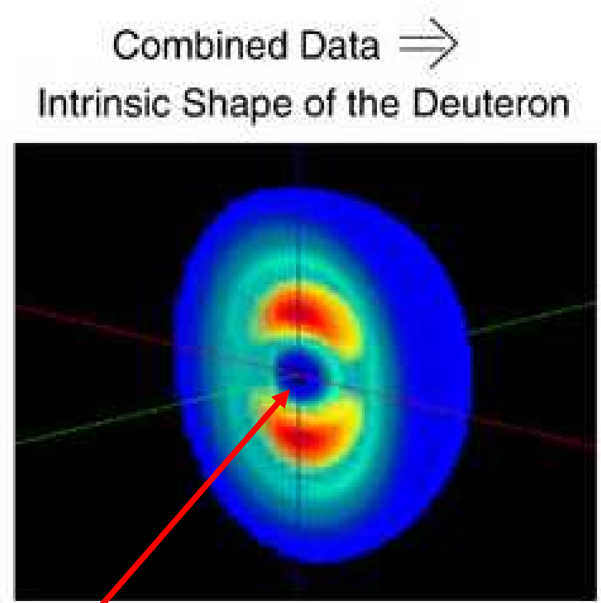
Important because:

- deuterium is the lightest nucleus and the only bound N-N state
- testing ground for state-of-the art models of the N-N interaction.

Electron scattering measurements:



Because the deuteron has spin 1, there are 3 form factors to describe elastic scattering: the "charge" (G_c), "electric quadrupole" (G_q) and "magnetic (G_M) form factors. (JLab data)



Interesting feature: strong attractive np force, but a void in the center – the deuteron is hollow! ... Why?

Interpretation of quantum numbers: $J^{\pi}, I \; = \; 1^{+}, \; 0$

$$\vec{S} \;\equiv\; \vec{S}_n + \vec{S}_p$$

$$\vec{J} \;=\; \vec{S} + \vec{L} \;=\; \vec{1}$$

$$\pi \;=\; (+)\,(+)\,(-1)^{L} \;=\; + \;\Rightarrow\; L \;=\; 0, 2, 4 \;\; .....$$

- Of the possible quantum numbers, L = 0 has the lowest energy, so we expect the ground state to be L = 0, S = 1 (the deuteron has no excited states!)
- The nonzero electric quadrupole moment suggests an admixture of L = 2 (more later!)

introduce **Spectroscopic Notation:** $^{2S+1}L_J$

with naming convention: L = 0 is an S-state, L = 1 is a P-state, L = 2: D-state, etc...

$\rightarrow$ the deuteron configuration is primarily $^{3}S_{1}$

$$\vec{I} = \vec{\frac{1}{2}} + \vec{\frac{1}{2}} \quad \Rightarrow \quad I = 0,\ 1$$

The total wavefunction for two identical Fermions has to be antisymmetric w.r.to particle exchange:

$$\Psi_{total} = \psi_{space} \times \phi_{spin} \times \chi_{isospin}$$

Central force problem:

$$\psi_{space}(r,\theta,\phi) = f(r)\ Y_{LM}(\theta,\phi)$$

with symmetry $(-1)^L$ given by the spherical harmonic functions

Spin and Isospin configurations:

S = 0 and I = 0 are antisymmetric ;

S = 1 and I = 1 are symmetric

$^3S_1$ state can only be I = 0 !

Magnetic Moment: $\mu_d = 0.857\ \mu_N$

In general, the magnetic moment is a quantum-mechanical vector; it must be aligned along the "natural symmetry axis" of the system, given by the total angular momentum:

$$\vec{\mu} \sim \vec{J}$$

But we don't know the direction of $\vec{J}$, only its "length" and z-projection as expectation values:

$$\left\langle J^2 \right\rangle = J\,(J+1)\,; \qquad \left\langle J_z \right\rangle = m_J = (-J \ldots. + J)$$

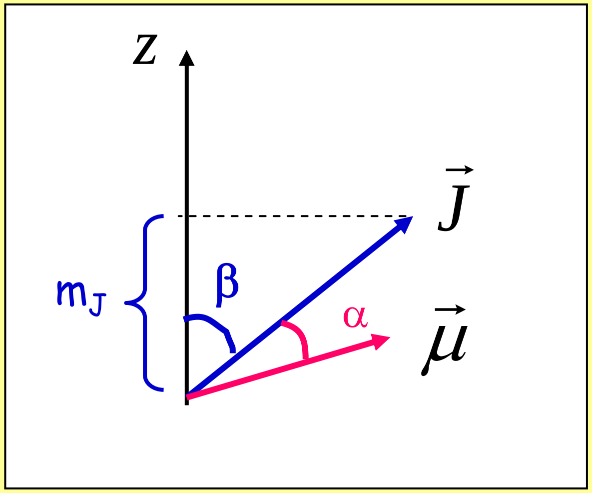
in a magnetic field, the energy depends on m_J via $\Delta E = -\left\langle \vec{\mu} \bullet \vec{B} \right\rangle \equiv -g_J\, m_J\, B\, \mu_N$

Strategy: we will define the magnetic moment by its maximal projection on the z-axis, defined by the direction of the magnetic field, with $m_J = J$

$$\mu \equiv \left\langle \vec{\mu} \bullet \hat{z} \right\rangle \Big|_{m_J = J} = g_J\, J\, \mu_N$$

Use expectation values of operators to calculate the result.....

Calculation of μ :



$$\mu \equiv \left\langle \vec{\mu} \bullet \hat{z} \right\rangle \Big|_{m_J = J} = g_J \; J \; \mu_N$$

Subtle point: we have to make two successive projections to evaluate the magnetic moment according to our definition, and the spin and orbital contributions enter with different weights.

1. Project onto the direction of J:

$$\vec{\mu} \bullet \hat{J} = \mu \cos\alpha = \frac{\vec{\mu} \bullet \vec{J}}{\sqrt{J(J+1)}}$$

2. Project onto the z-axis with m_J = J:

$$\cos\beta = \frac{m_J}{|J|} = \frac{J}{\sqrt{J\,(J+1)}}$$

$$\boxed{\mu \equiv g_J \; J \; \mu_N = \mu \cos\alpha \cos\beta = \left\langle \vec{\mu} \bullet \vec{J} \right\rangle \frac{1}{(J+1)}}$$

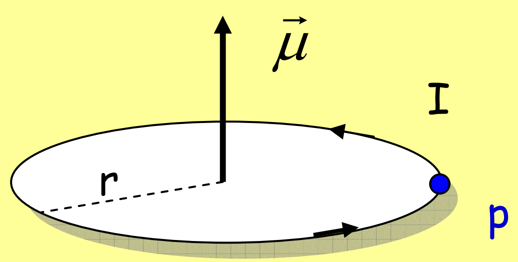
Next, we need to figure out the operator for $\vec{\mu}$

We already know the intrinsic magnetic moments of the proton and neutron, so these must correspond to the spin contributions to the magnetic moment operator:

$$\mu_p = +2.79\,\mu_N = g_{s,p}\,S\,\mu_N \Rightarrow g_{s,p} = +5.58$$

$$\mu_n = -1.91\,\mu_N = g_{s,n}\,S\,\mu_N \Rightarrow g_{s,n} = -3.83$$

For the orbital part, there is a contribution from the proton only, corresponding to a circulating current loop (semiclassical sketch, but the result is correct)



$$\vec{\mu} = I\,\pi\,r^2\,\hat{\ell} = g_\ell\,\vec{\ell}\,\mu_N\,, \quad g_\ell = 1$$

For the deuteron, we want to use the magnetic moment operator:

$$\vec{\mu} = \left(g_{s,p}\,\vec{S}_p + g_{s,n}\,\vec{S}_n + \vec{L}_p \right) \mu_N$$

Details:

1. $\vec{L}_p = \frac{1}{2}\vec{L}$ because $m_n \cong m_p$, and L is the total orbital angular momentum !

2. L = 0 in the "S-state" ($^3S_1$) but we will consider also a contribution from the "D-state" ($^3D_1$) as an exercise

3. The proton and neutron couple to S = 1, and the deuteron has J = 1

$$\mu = \frac{1}{2}\left\langle \vec{\mu} \bullet \vec{J} \right\rangle = \frac{1}{2}\left\langle \left(g_{s,p}\,\vec{S}_p + g_{s,n}\,\vec{S}_n + \frac{1}{2}\vec{L} \right) \bullet \vec{J} \right\rangle \mu_N$$

Trick: use $\vec{S}_n + \vec{S}_p = \vec{S}$ and write the **operator** as:

$$\vec{\mu} = \left(\frac{1}{2}\,(g_{s,p} + g_{s,n})\,\vec{S} + \cancel{\frac{1}{2}(g_{s,p} - g_{s,n})\,(\vec{S}_p - \vec{S}_n)} + \frac{1}{2}\vec{L} \right) \mu_N$$

But the proton and neutron spins are aligned, and $\left\langle \vec{S}_p \bullet \vec{J} \right\rangle = \left\langle \vec{S}_n \bullet \vec{J} \right\rangle$

so the second term has to give zero!

So, effectively we can write for the deuteron:

$$\mu = \tfrac{1}{4} \left\langle \left((g_{s,p} + g_{s,n}) \ \vec{S} + \ \vec{L} \ \right) \bullet \vec{J} \right\rangle \ \mu_N$$

Trick for expectation values:

$$\vec{J} = \vec{L} + \vec{S} \, ; \qquad \left\langle J^2 \right\rangle = J\,(J+1)$$

$$\left\langle J^2 \right\rangle = \left\langle (\vec{L}+\vec{S}) \bullet (\vec{L}+\vec{S}) \right\rangle = \left\langle L^2 + S^2 + 2\vec{L} \bullet \vec{S} \right\rangle$$

$$\left\langle \vec{S} \bullet \vec{J} \right\rangle = \left\langle \vec{S} \bullet \vec{L} + \vec{S} \bullet \vec{S} \right\rangle = \tfrac{1}{2} \left\langle J^2 - L^2 - S^2 \right\rangle + \left\langle S^2 \right\rangle = \tfrac{1}{2} \left\langle J^2 - L^2 + S^2 \right\rangle$$

$$\left\langle \vec{L} \bullet \vec{J} \right\rangle = \left\langle \vec{L} \bullet \vec{L} + \vec{L} \bullet \vec{S} \right\rangle = \tfrac{1}{2} \left\langle J^2 + L^2 - S^2 \right\rangle$$

$$\mu\,(^3S_1) = \tfrac{1}{2} (g_{s,p} + g_{s,n}) \ \mu_N = \mu_p + \mu_n = 0.880 \ \mu_N$$

$$\mu\,(^3D_1) = \tfrac{1}{4} \Big(3 - (g_{s,p} + g_{s,n}) \Big) \mu_N = 0.310 \ \mu_N$$

Comparison to experiment:

$$\mu_d = 0.857 \ \mu_N$$

This is intermediate between the S-state and D-state values:

$$\mu\,({}^3S_1) = 0.880 \ \mu_N$$

$$\mu\,({}^3D_1) = 0.310 \ \mu_N$$

Suppose the wave function of the deuteron is a linear combination of S and D states:

$$\left|\psi_d\right\rangle = a \left|{}^3S_1\right\rangle + b \left|{}^3D_1\right\rangle \quad with \quad a^2 + b^2 = 1$$

Then we can adjust the coefficients to explain the magnetic moment:

$$\mu_d = (1-b^2)\ \mu\,({}^3S_1) + b^2\ \mu\,({}^3D_1)$$

b^2 = 0.04, or a 4% D-state admixture accounts for the magnetic moment !